# Effects of Superheated Vapor and Noncondensable Gases on Laminar Film Condensation

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**A boundary-layer analysis of laminar film condensation on a vertical plate has been carried out to study the effects of superheated vapor and noncondensable gases. For a pure, superheated vapor the effects of superheating on the surface heat transfer are accounted for by a simple modification of previously available results for saturated vapors. For a given temperature difference between the plate surface and the liquid-vapor interface, superheating increases the heat transfer to the surface, but only to a modest extent for most practical situations. Although free convection was not included in the analysis, its role has been estimated to be small. For noncondensable gases the analysis shows that the presence of a few per cent of noncondensable in the bulk of the vapor causes a great reduction in the surface heat transfer. The fact that the predicted reduction is substantially larger than that found experimentally indicates that free convection, which is not included in the analysis, plays an important role when noncondensables are present.**

The theory of gravity-induced laminar film condensation on a vertical surface has seen several new developments in recent years. The original theory as given by Nusselt (*1*) in 1916 consisted of simple force and heat balances within the condensate film, and inertia forces, energy convection, and interfacial shear were not considered. The effects of energy convection were studied by Bromley (*2*) and Rohsenow (*3*). Later the problem was brought within the framework of boundary-layer theory by Sparrow and Gregg (*4, 5*), and inertia forces and energy convection were included. Very recently (*6, 7*) the analysis has been extended to include interfacial drag, and the flow induced within the vapor has been determined (*6*).

The foregoing analyses have been carried out under the conditions that the vapor is at its saturation temperature and that noncondensable gases are absent. In this report initial consideration will be given to film condensation of a superheated pure vapor. The effect of noncondensables will also be discussed.

The physical situation to be studied is pictured schematically in Figure 1. A cooled vertical surface has a uniform temperature $T_w$, while the interface between the liquid condensate and the vapor is at the saturation temperature $T_i$ corresponding to the local vapor pressure. The bulk of the vapor is at $T_\infty$. Only gravity-induced flow is considered; that is there is no forced convection in the bulk of the vapor. Fluid motions within the condensate arise owing to body forces. In the vapor the fluid motions arise from two causes. First, owing to the shear forces at the liquid-vapor interface, the vapor is swept along by the downward-flowing liquid. Second, owing to the density difference between the interface and the bulk of the vapor, a free convection flow is set up. The vapor motions associated with each of these mechanisms operate additively to establish a flow pattern which is directed inward toward the interface and downward. The analysis is carried out for the situation where the flow is laminar in both liquid and vapor. Additionally the interface between liquid and vapor is postulated to be smooth and ripple free.

In the main body of the report consideration will be given to pure, superheated vapors. In the final section the effects of noncondensable gases will be analyzed.

The convective heat transfer problem in the vapor associated with the combined flow due to free convection and interfacial shear is very formidable indeed. As a first step the authors propose to consider in detail only the effects of the vapor flow due to interfacial shear. For a pure vapor it is demonstrated that the effects of superheating on the heat transfer at the condenser surface can be accounted for in a remarkably simple way. The role of free convection is estimated by a simple additive process, and from this one is led to believe that free convection effects do not markedly alter the heat transfer predictions of the present analysis. For the situation where noncondensable gases are present, it is analytically demonstrated from first principles that the heat transfer is drastically reduced by the presence of a few per cent of noncondensable in the bulk of the vapor. However free convection may play an important role in this case, and further work appears to be needed in order to establish more accurate quantitative predictions.

## ANALYSIS FOR PURE, SUPERHEATED VAPORS

In formulating any convective heat transfer problem it is necessary to have either a prior or simultaneous knowledge of the velocity distribution within the fluid. Consequently this article begins with a brief review of the velocity problem.

### Velocity Problem

Under the assumption of constant fluid properties the velocity results can be obtained without any recourse to the temperature problem. Inasmuch as the constant property assumption has been made in reference *6* and is being retained here, one can use the velocity results of the reference. There the two-phase flow problem was formulated by writing differential equations expressing conservation of mass and momentum in the liquid and in the vapor. At the liquid-vapor interface $y = \delta$, it was required that continuity be satisfied by the tangential velocity, the mass flow, and the shear stress. It was found that by introducing similarity variables the problem could be formulated as an exact boundary-layer solution.

From the velocity analysis it is useful for present purposes to record the transformed variables in terms of which the solutions were obtained. Liquid layer

$$\eta_L = \frac{c_L y}{x^{1/4}}, \quad c_L = \left[\frac{g(\rho_L - \rho_v)}{4\nu_L{}^2\rho_L}\right]^{1/4} \tag{1a}$$

$$u = 4\nu_L\, c_L{}^2\, x^{1/2}\, F'(\eta_L),$$

$$v = \nu_L\, c_L\, x^{-1/4}[\eta_L F'(\eta_L) - 3F(\eta_L)] \tag{1b}$$

Vapor layer

$$\eta_v = \frac{c_v(y-\delta)}{x^{1/4}},$$

$$c_v = \left[\frac{g(\rho_L - \rho_v)}{4\,\nu_v{}^2\,\rho_L}\right]^{1/4} \tag{2a}$$

$$u = 4\nu_v c_v{}^2\, x^{1/2} f'(\eta_v),$$

$$v = \nu_v\, c_v\, x^{-1/4}\,[(c_v\, y/x^{1/4}) \quad f'(\eta_v) - 3f(\eta_v)] \tag{2b}$$

where primes denote differentiation with respect to the similarity variables $\eta_L$ and $\eta_v$. The functions $F(\eta_L)$ and $f(\eta_v)$ are essentially dimensionless stream functions. Additionally the local mass flux $\dot{m}$ crossing the liquid-vapor interface ($y = \delta$) is given by

$$\dot{m} = \left[\rho\left(u\frac{d\delta}{dx} - v\right)\right]_\delta$$

$$= 3\mu_L c_L x^{-1/4}\, F(\eta_{L\delta}) \tag{3}$$

where $F(\eta_{L\delta})$ denotes the value of $F$ at the interface $\eta_L = \eta_{L\delta}$. The solution also indicates that the film thickness $\delta$ varies as $\delta \sim x^{1/4}$. So, the interface is defined by the condition $\eta_{L\delta}$ = constant. However it is important to note that $\delta$, and hence $\eta_{L\delta}$, is not known apriori but rather would be one of the results to be found.

The approach toward solving the velocity problem was to arbitrarily select a value of $\eta_{L\delta}$ and then to numerically obtain a solution for $F(\eta_L)$ and $f(\eta_v)$ corresponding to this choice. Then, by means of the energy equation, the temperature conditions to which the $\eta_{L\delta}$ corresponded were determined. The authors will use the same approach here, but now, with a superheated vapor, a somewhat different energy formulation is required.

### Temperature Problem

Attention is given first to the liquid layer. One may define a dimensionless temperature $\sigma_L$ as

$$\sigma_L(\eta_L) = \frac{T - T_i}{T_w - T_i} \tag{4}$$

where $\sigma_L$ depends only on the similarity variable $\eta_L$. With this definition the energy equation in the liquid reduces to that treated in reference 6. From the solutions given there the local heat transfer $q$ at the plate surface is found by applying Fourier's Law $q = -k(\partial T/\partial y)_{y=0}$. Substituting the transformed variables of Equations (1*a*) and (4) and introducing the local heat transfer coefficient and local Nusselt number one gets

$$h = \frac{q}{T_i - T_w} \qquad N_{Nu_x} = \frac{hx}{k}$$

The dimensionless representation of the local heat transfer is

$$N_{Nu_x}\left[\frac{4\,\rho_L\,\nu_L{}^2}{g(\rho_L - \rho_v)x^3}\right]^{1/4} = -\left(\frac{d\sigma_L}{d\eta_L}\right)_{\eta_L=0} \tag{5}$$

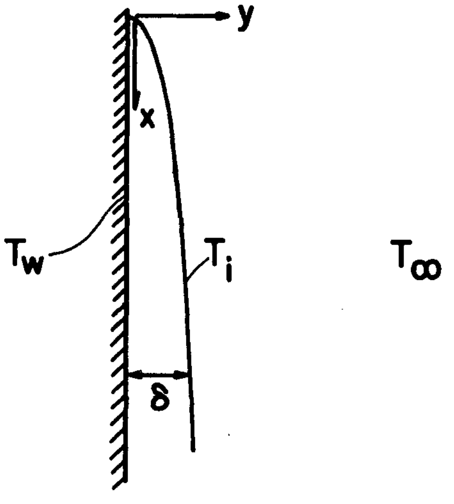

Fig. 1. Physical model and coordinate system.

The derivative $(d\sigma_L/d\eta_L)_{\eta_L=0}$ has been calculated in reference 6 as a function of the Prandtl number $Pr_L = (c_p\mu/k)_L$ and of the dimensionless film thickness $\eta_{L\delta}$ by solving the energy equation for the liquid. However since the film thickness remains to be found, the heat transfer result expressed by Equation (5) is not complete.

To determine the film thickness one must solve the energy equation for the vapor and also take account of the energy continuity condition at the liquid-vapor interface. One begins by writing the energy equation for the vapor:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\frac{k}{\rho c_p}\right)_v \frac{\partial^2 T}{\partial y^2} \tag{6}$$

Introducing a dimensionless temperature $\sigma_v$ as

$$\sigma_v(\eta_v) = \frac{T - T_\infty}{T_i - T_\infty} \tag{7}$$

and substituting the transformed variables of Equations (2*a*) and (2*b*) one gets the energy equation

$$\sigma_v'' + 3\,Pr_v\, f\, \sigma_v' = 0 \tag{8}$$

The boundary conditions that $T = T_i$ at $y = \delta$ and that $T \to T_\infty$ as $y \to \infty$ become

$$\sigma_v(0) = 1, \qquad \sigma_v(\infty) = 0 \tag{9}$$

Although the temperature profile may be of some interest, the most useful result in the heat transfer calculation is the heat flux conducted across the interface, and this is proportional to $(d\sigma_v/d\eta_v)$ at $\eta_v = 0$. It may be easily verified that a solution for the derivative which satisfies Equations (8) and (9) is

$$\left(\frac{d\sigma_v}{d\eta_v}\right)_0 = -\frac{1}{\int_0^\infty \left[e^{-3Pr_v \int_0^{\eta_v} f\, d\eta_v}\right] d\eta_v} \tag{10}$$

To obtain numerical results it remains to integrate the velocity function $f$. The distribution of $f$ depends upon two parameters: the property ratio $[(\rho\mu)_L/(\rho\mu)_v]^{1/2}$ and $\eta_{L\delta}$. However if one restricts himself by the condition

$$[(\rho\mu)_L/(\rho\mu)_v]^{1/2} \sim 150 \text{ or greater} \tag{11}$$

then a study of the velocity solutions of reference 6 reveals that for practical values of $\eta_{L\delta}$.

$$\left(\frac{d\sigma_v}{d\eta_v}\right)_0 \cong -3\,Pr_v\, f(0) \tag{10a}$$

The restriction embodied in Equation (11) is not severe. For example for condensing steam at 1 atm. $[(\rho\mu)_L/(\rho\mu)_v]^{1/2}$ is about 200, while at 1 in. of mercury, the ratio is 2,000. For mercury and ethanol at 1 atm. $[(\rho\mu)_L/(\rho\mu)_v]^{1/2}$ is 700 and 150, respectively. Consequently, except for condensation at pressures above 1 atm., Equation (10*a*) can be regarded as a very accurate representation of the (dimensionless) heat conducted across the interface from the vapor into the liquid.

### Numerical Results for the Nusselt Number

It now only remains to tie the temperature problem in the liquid and vapor together by requiring that energy be conserved at the interface. Such an energy balance yields

$$\left(k\frac{\partial T}{\partial y}\right)_L = \dot{m}\, h_{fg} + \left(k\frac{\partial T}{\partial y}\right)_v \tag{12}$$

The left-hand side is the heat conducted away into the liquid. The first term on the right represents the energy liberated as latent heat, while the last term is the heat conducted to the interface through the vapor. Substi-

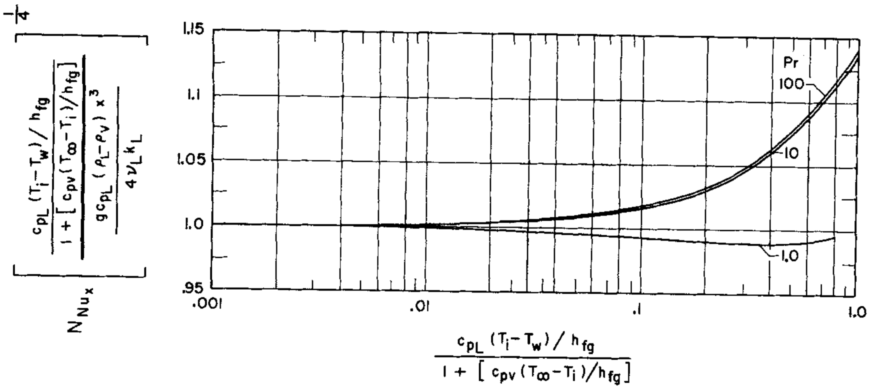

**Fig. 2. Local heat transfer results for high Prandtl number range ($h_{av}$ = 4/3$h$).**

tuting for the mass flow $\dot{m}$ from Equation (3) and introducing transformed variables from Equations (1a), (2a), (4), and (7) one gets for the interfacial energy balance

$$\frac{c_{pL}\,(T_w - T_i)}{h_{fg}\,Pr_L}\left[\frac{d\sigma_L}{d\eta_L}\right]_{\eta_{L\delta}} = 3\,F(\eta_{L\delta}) + \frac{c_{pv}\,(T_i - T_\infty)}{h_{fg}\,Pr_v}\left[\frac{d\sigma_v}{d\eta_v}\right]_0\left[\frac{(\rho\mu)_v}{(\rho\mu)_L}\right]^{1/2} \qquad (12a)$$

Next eliminating $[d\sigma_v/d\eta_v]_0$ according to Equation (10a) and noting (as shown in reference 6) that

$$f(0)\left[\frac{(\rho\mu)_v}{(\rho\mu)_L}\right]^{1/2} = F(\eta_{L\delta}) \qquad (13)$$

is a necessary condition for mass conservation, one gets for Equation (12a)

$$\frac{c_{pL}\,(T_i - T_w)/h_{fg}}{1 + [c_{pv}\,(T_\infty - T_i)/h_{fg}]} = \frac{3\,Pr_L\,F(\eta_{L\delta})}{-\,[d\sigma_L/d\eta_L]_{\eta_{L\delta}}} \qquad (14)$$

For prescribed values of dimensionless film thickness $\eta_{L\delta}$ and liquid Prandtl number $Pr_L$ the right side of Equation (14) is known from the velocity and temperature solutions. Next when one reconsiders the Nusselt number expression

$$N_{Nu_x}\left[\frac{4\,\rho_L\nu_L^2}{g(\rho_L - \rho_v)\,x^3}\right]^{1/4} = -\left[\frac{d\sigma_L}{d\eta_L}\right]_{\eta_L=0} \qquad (5)$$

One notes that here also the right side is known as soon as $Pr_L$ and $\eta_{L\delta}$ are prescribed. Hence by evaluating both Equations (14) and (5) over a range of $Pr_L$ and $\eta_{L\delta}$ one can find the relationship between $N_{Nu_x}[4\rho_L\nu_L^2/g(\rho_L - \rho_v)\,x^3]^{1/4}$ and

$$\frac{c_{pL}(T_i - T_w)/h_{fg}}{1 + [c_{pv}\,(T_\infty - T_i)/h_{fg}]} \qquad (15)$$

This is exactly the same procedure as was carried out for the case of saturated vapor (reference 6), except that there the simpler group $c_{pL}(T_i - T_w)/h_{fg}$ appeared in place of the parameter (15). Consequently one can directly employ the heat transfer results of reference 6, with the change that $c_{pL}(T_i - T_w)/h_{fg}$ is replaced by the parameter (15). The heat transfer results thus obtained are exhibited on Figures 2 and 3, respectively, for the high and low liquid Prandtl number ranges.

By inspection of the figures it is seen that for very small values of the abscissa variable (very thin films) the results can be expressed as

$$N_{Nu_x}\left[\frac{c_{pL}\,(T_i - T_w)/h_{fg}}{gc_{pL}\,(\rho_L - \rho_v)x^3/4\nu_L\,k_L}\right]^{1/4} = \left[1 + \frac{c_{pv}(T_\infty - T_i)}{h_{fg}}\right]^{1/4} \qquad (16)$$

In this form the second term on the right represents a correction factor due to the superheating of the vapor. For example if $c_{pv}(T_\infty - T_i)/h_{fg} = 0.5$ (a very large superheating), the correction would be about 10%. This is a modest effect for such a large superheat. For abscissa values which are not small further study of Figure 2 reveals that for high Prandtl numbers the correction term is somewhat less than that given by Equation (16). On the other hand from Figure 3 it is seen that the correction for low Prandtl number liquids can be much larger than that of Equation (16).

### Experimental Verification

It is of course desirable to test any theory by comparison with the results of experiment. Now among condensation experiments there is a wide spread in data. Even for the case of saturated vapor there are experimental Nusselt numbers which fall well below theory and some which fall well above theory. With this in mind it would appear that a proper test of the theory with superheat would be comparison with a set of experiments, performed on a given apparatus, in which the superheat is varied systematically starting from the case of saturated vapor. Then the effects of superheat for that apparatus could be determined as a ratio of the Nusselt number with superheat to that without superheat. It is felt that using the saturation data for a given apparatus as a standard would tend to remove any peculiarities arising from that apparatus. The experimentally determined Nusselt number ratios just described would then provide a meaningful comparison with theory. It is believed that superheat data which are not standardized (by the saturation data) for the particular apparatus would not provide a proper test of the theory. The authors have been unable

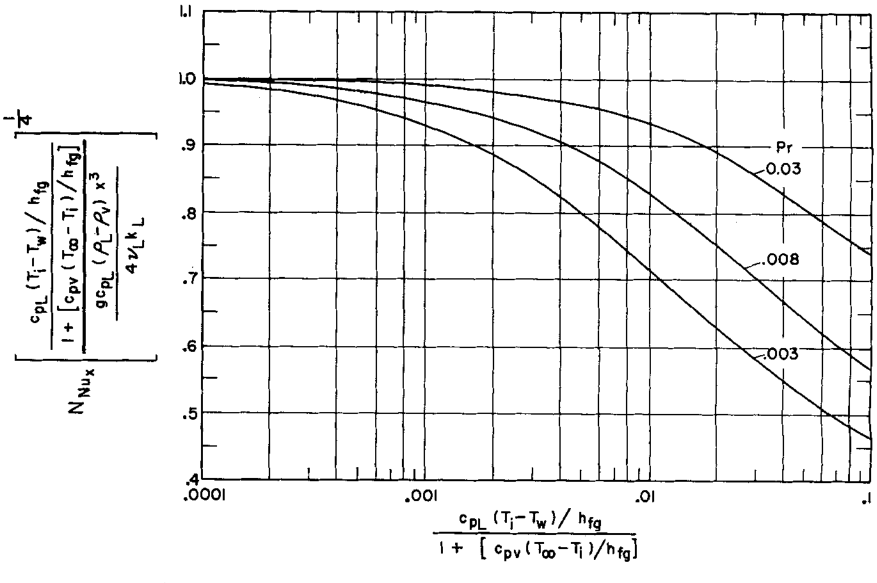

**Fig. 3. Local heat transfer results for low Prandtl number range ($h_{av}$ = 4/3$h$).**

to find any systematic experimental study of the type described above.

### Free Convection Effects

It is of interest to obtain an approximate estimate as to the effects of free convection on the predictions presented here. To do this one returns to the over-all energy balance (12) and adds in a term $h(T_\infty - T_i)$ to characterize the free convection heat transfer to the interface. The coefficient $h$ will be taken as that for the vertical isothermal plate, and it is believed that this will overestimate the free convection effect. When one characterizes the free convection in this way, there appears in Equation (12*a*) after some rearrangement an additional term:

$$\frac{c_{pv}\,(T_\infty - T_i)}{h_{fg}\;Pr_v}\left[\frac{(\rho\mu)_v}{(\rho\mu)_L}\right]^{1/2}\left\{[\beta(T_\infty - T_i)]^{1/4}\,\frac{N_{Nu_x}}{(N_{Gr_x}/4)^{1/4}}\right\} \tag{17}$$

Then to compare the relative energy transport by free convection and shear-induced flow one takes the ratio of (17) to the last term of Equation (12*a*), giving

$$\frac{[\beta(T_\infty - T_i)]^{1/4}}{f(0)}\left\{\frac{N_{Nu_x}/\left(\dfrac{N_{Gr_x}}{4}\right)^{1/4}}{3\,Pr_v}\right\} \tag{18}$$

in which Equation (10*a*) has also been used. For vapors $Pr \sim 1$, and the quantity in braces has a value (reference *8*) of about 0.20. Additionally $\beta \approx \frac{1}{T_\infty}$, where $T_\infty$ must be expressed in degrees absolute, and in practice it would not be expected that $[\beta(T_\infty - T_i)]^{1/4}$ would exceed 0.75. With these numerical values the ratio of energy transports as expressed by Equation (18) becomes $0.15/f(0)$, where the numerical value of $f(0)$ depends on $\eta_{L\delta}$ and the property ratio $[(\rho\mu)_L/(\rho\mu)_v]^{1/2}$. For the smallest practical values of $\eta_{L\delta}$ and a $(\rho\mu)^{1/2}$ ratio of 200 the energy transport ratio of Equation (18) is perhaps 1/3 and decreases to 1/15 as the ratio increases to 1,000. For thicker films still within the practical range the energy transport ratio will certainly drop to below 0.05. Consequently based on these arguments the free convection energy transport would be expected to be substantially less than that due to the shear-induced flow.

The effect of the free convection on the results obtained here can be gauged by inserting a constant (> 1) as a multiplier of the factor $c_{pv}(T_\infty - T_i)/h_{fg}$ in the denominator of expression (15). The value of the constant would be equal to unity plus the value of the energy transport ratio (18). The same multiplier of $c_{pv}(T_\infty - T_i)/h_{fg}$ would appear within the brackets on the right side of the thin film Equation (16). From the prior discussion such a multiplier could deviate appreciably from unity only for very thin films. However by inspection of Equation (16) it is easy to see that even under these circumstances the presence of the constant unity within the bracket on the right side and the quarter root of the bracket will both work to dilute the free convection effects. Thus one is persuaded to believe that the predicted effect on heat transfer of superheating the vapor will not be much modified by free convection, provided that noncondensable gases are absent.

## NONCONDENSABLE GASES

In this section of the paper it will be demonstrated that the presence of small amounts of noncondensable gases will have an important effect in decreasing the condensation heat transfer. Prior analytical studies which have been concerned with the effects of noncondensables are reported in references 9 through *13*. The studies of Van der Held (*9*) and Colburn and Drew (*10*) are based on a one-dimensional diffusional flow in the vapor-gas mixture. Such an analytical model cannot be fully predictive, since momentum and energy conservation are not considered. The analysis of Baer and McKelvey (*11*) assumes that there is an additional resistance at the liquid-vapor interface which is very sensitive to the presence of noncondensable gases, but that the diffusional resistance in the gas-vapor mixture is negligible. This model is currently still under debate (*12, 13*). Additionally the theory requires a condensation coefficient which must be determined empirically.

In the present analysis the problem will be attacked from first principles by including momentum and energy conservation along with diffusion. In this first step toward a fully-predictive theory, free convection within the vapor will not be included. However since one is not able to prove that free-convection effects are necessarily small in the presence of noncondensable gases, the conclusions which will be drawn are regarded as qualitative rather than quantitative.

In this spirit the authors propose to carry out the analysis on the basis of constant fluid properties for the gas-vapor mixture as a whole.* Previous experience with forced convection mass transfer problems has shown that qualitative trends deduced from the constant property analysis are verified by more complete variable property analysis. The condensing vapor is denoted as fluid 1, the noncondensable gas as fluid 2, and the respective mass fractions $W_1$ and $W_2$ are defined by

$$W_1 = \frac{\rho_1}{\rho} \qquad W_2 = \frac{\rho_2}{\rho} \tag{19}$$

Further, since $\rho_1 + \rho_2 = \rho$, then

$$W_1 + W_2 = 1 \tag{20}$$

From mass conservation there may be derived the following diffusion equation (*15*):

$$u\frac{\partial W_1}{\partial x} + v\frac{\partial W_1}{\partial y} = D\frac{\partial^2 W_1}{\partial y^2} \tag{21}$$

Here D represents a binary diffusion coefficient. Next a dimensionless mass fraction may be introduced by

$$\Phi(\eta_v) = \frac{W_1 - W_{1\infty}}{W_{1i} - W_{1\infty}} \tag{22}$$

where $W_{1i}$ and $W_{1\infty}$ are respectively the interfacial and free stream mass fractions of the condensing vapor. The transformation of variables as defined by Equations (2*a*) and (2*b*) may be applied to the diffusion Equation (21), giving

$$\Phi'' + 3\,N_{Sc}\,f\Phi' = 0 \tag{23a}$$

$N_{Sc} = \nu/D$ is the Schmidt number. The boundary conditions that $W_1 = W_{1i}$ at $y = \delta$ and $W_1 = W_{1\infty}$ as $y \rightarrow \infty$ may be rewritten in terms of the new variables as

$$\Phi(0) = 1, \qquad \Phi(\infty) = 0 \tag{23b}$$

It is thus seen that the diffusion problem defined by Equation (23) has the same form as the temperature problem (8) and (9). The solution for $[d\Phi/d\eta_v]_0$ may be taken directly from Equation (10), replacing $Pr$ by $Sc$.

In addition to satisfying the diffusion equation the mass fraction distribution must also satisfy the condition that the liquid-vapor interface is impermeable to the noncondensable gas 2. In analogy with Equation (3) the mass flux of component 2 at the interface is

$$m_2 = \left[\rho_2\left(u_2\frac{d\delta}{dx} - v_2\right)\right]_\delta \tag{24}$$

where now $u$ and $v$ include both convective (sub *c*) and diffusional (sub *d*) velocity components:

$$u_2 = u_c + u_d = u_c - \frac{D}{W_2}\frac{\partial W_2}{dx} \tag{25a}$$

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* This excludes a diffusive flow of sensible heat (Aukerman effect, reference 14).

$$v_2 = v_c + v_d = v_c - \frac{D}{W_2}\frac{\partial W_2}{dy} \tag{25b}$$

But from (20)
$W_2 = 1 - W_1$,

$$\frac{\partial W_2}{\partial x} = -\frac{\partial W_1}{\partial x}, \quad \frac{\partial W_2}{\partial y} = -\frac{\partial W_1}{\partial y} \tag{26}$$

Then introducing Equations (25) and (26) and setting $\dot{m}_2 = 0$, one gets for Equation (24)

$$\left[ u_c \frac{d\delta}{dx} - v_c + \frac{D}{1 - W_1} \left( \frac{\partial W_1}{\partial x}\frac{d\delta}{dx} - \frac{\partial W_1}{\partial y} \right) \right]_\delta = 0$$

By boundary-layer assumptions the first term in the parentheses can be neglected compared with the second. Then making use of the transformation (2*a*) of the definition of $\Phi$ (22) and of the convective velocities from (2*b*) one obtains

$$\frac{W_{1\infty} - W_{1i}}{1 - W_{1i}} = \frac{3\,N_{Sc}\,f\,(0)}{-[d\Phi/d\eta_v]_0} \tag{27}$$

The first matter about which one wants to inquire is whether it is possible to maintain the condensation rates found in pure vapors when noncondensable gases are present. For the range of film thicknesses $\eta_{L\delta}$ which are encountered in the condensation of pure vapors the derivative $[d\Phi/d\eta_v]_0$ deviates from $[-3N_{Sc}f(0)]$ by only a fraction of a per cent, provided that the $\rho\mu$ ratio is restricted as noted in Equation (11). Then from Equation (27) it follows that $W_{1\infty}$ deviates from unity by an even smaller fraction of a per cent. Consequently it may be concluded that condensation rates found for pure vapors can be maintained only if the noncondensables in the bulk of the vapor are restricted to minute amounts. This qualitative statement is in accord with experiment.

Next one wants to inquire as to the predicted decrease in heat transfer if larger amounts of noncondensables are present. Consider a situation where saturated steam at 195°F. ($= T_\infty$) is to be condensed on a wall at 140°F. In one case the vapor is pure, while in a second case the mass fraction of air in the bulk of the fluid is 2%; that is $W_{1\infty} = 0.98$. In the absence of the noncondensable the interface temperature $T_i$ is also 195°, and hence the temperature difference across the liquid film $T_i - T_w = 55$°F. With this information the heat transfer may be determined from Figure 2. For the case where the noncondensable is present, the heat calculation first requires that the interface mass fractions be determined. For a Schmidt number of 0.5 for the steam-air system the derivative $[d\Phi/d\eta_v]_0$ has been calculated from Equation (10) by numerically integrating the $f$ values from reference 6. From this, in conjunction with Equation (27), the vapor mass fraction at the interface $W_{1i}$ is found to be 19%. Consequently the mass fraction of the noncondensable is 81%. The corresponding interface temperature is 140.3°F., and this gives a driving force $T_i - T_w$ for heat transfer across the liquid film of 0.3°F. Again from Figure 2 it is found that the predicted heat transfer $q$ in the presence of the noncondensable is decreased to 1/46 of its value for the pure vapor. This decrease is substantially greater than that measured by Othmer (as reported by McAdams in reference *16*). A significant part of the discrepancy is likely due to the fact that free convection has not been included in the analysis. The role of free convection would be to aid in the removal of noncondensables by increasing the flow in the direction parallel to the plate surface.

The discussion given here, while qualitatively indicating the importance of noncondensables, also serves as a guidepost for future work. Clearly the effects of free convection must be included in the analysis in order to achieve more accurate quantitative predictions.

## NOTATION

$c$ = dimensional constant, see Equations (1*a*) and (2*a*)
$c_p$ = specific heat at constant pressure
$D$ = diffusion coefficient
$F$ = dimensionless velocity variable for liquid
$f$ = dimensionless velocity variable for vapor
$N_{Gr_x}$ = Grashof number, $g\beta(T_\infty - T_i)\,x^3/\nu^2$
$g$ = acceleration of gravity
$h$ = local heat transfer coefficient, $q/(T_i - T_w)$
$h_{fg}$ = latent heat of condensation
$k$ = thermal conductivity
$\dot{m}$ = mass flow at interface
$N_{Nu_x}$ = Nusselt number, $hx/k$
$Pr$ = Prandtl number, $c_p\mu/k$
$q$ = local heat transfer rate per unit area at plate surface
$N_{Sc}$ = Schmidt number, $D/\nu$
$T$ = temperature; $T_w$, surface temperature; $T_i$, interface temperature; $T_\infty$, ambient vapor temperature
$u$ = velocity component in the $x$ direction
$v$ = velocity component in the $y$ direction
$W$ = mass fraction, $\rho_i/\rho$
$x$ = coordinate measuring distance from the leading edge
$y$ = coordinate measuring distance normal to the surface

**Greek Letters**

$\beta$ = thermal expansion coefficient, $-\frac{1}{\rho}\left[\frac{\partial\rho}{\partial T}\right]_p$
$\delta$ = film thickness
$\eta_L, \eta_v$ = similarity variables, see Equations (1*a*) and (2*a*)
$\eta_{L\delta}$ = dimensionless film thickness, $c_L\delta/x^{1/4}$
$\sigma_L, \sigma_v$ = dimensionless temperatures, see Equations (4) and (7)
$\mu$ = absolute viscosity
$\nu$ = kinematic viscosity
$\rho$ = density
$\Phi$ = dimensionless mass fraction, see Equation (22)

**Subscripts**

$L$ = liquid
$v$ = vapor
$\delta$ = at $y = \delta$
1 = condensable vapor
2 = noncondensable gas

## LITERATURE CITED

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